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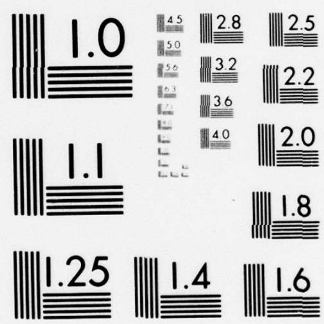

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HYPERSONIC AREA RULE

by

M. D. Ladyzhenskiy

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EDITED TRANSLATION

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Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
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Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
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М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

1925

HYPERSONIC AREA RULE

M. D. Ladyzhenskiy (Moscow)

Abstract

Assuming [2] that the entire mass of gas is concentrated in the infinitely thin layer adjacent to the shock wave, report [1] formulates the hypersonic area rule. According to this rule, during flow about thin nonaxisymmetrical bluff bodies with equal values of blunting resistance and identical laws of the change in the direction of the flow on the cross-sectional area, the shock wave surfaces, the laws of the change in pressure, and, consequently, also the drag acting on the body coincide. Here the surfaces of the shock waves are

axially symmetrical.

This study establishes the limits of applicability of the results in [1] and refines the hypersonic area rule by introducing the entropy layer.

1. Determination of the Limits of Applicability of the Results of [1]. As an example of using the hypersonic area rule [1], we will construct a body equivalent to a thin round cone, i.e., for which the blunting resistance and the law of the change in the flow direction of the cross-sectional area are identical to those of a cone. The cross section of the body is assumed to be shaped like an ellipse whose long semiaxis is equal to the radius of the shock wave and whose area is equal to that of the cross section of the round cone (Fig. 1). In other words, the eccentricity of the ellipse has the maximum possible value which still satisfies the requirement (condition (3) from [1]) in each cross section, in accordance with which the body should not go beyond the limits of the space bounded by the surface of the shock wave.

As [1] pointed out, the area rule can be combined with the law of similarity during flow about thin bluff bodies [2], as a result of which the dimensionless values which characterize the flow are

determined by two dimensionless parameters at a fixed value of the adiabatic index $\varkappa$: the known similarity parameter during flow about thin bluff bodies $K = M_\infty \tau$ and parameter $K_1 = (\pi / 2c_x S)^{1/2} L \tau^2$, which characterizes the blunting effect of equal order of magnitude to the square root of the ratio of the resistance of the body to the blunting resistance. Here $\tau \sim S_*^{1/2}/L$ — is the small dimensionless value which characterizes the thickness of the body; S* is a certain characteristic cross-sectional area of the body; L is the length of the body; and c_x, S are the blunting resistance coefficient and the maximum midsectional area of blunting, respectively.

Assuming that the effect of blunting can be replaced by an explosive effect at the front point of the body with energy equal to the blunting resistance, blunting is insignificant [1]. Thus, blunting area S is introduced to express K_1 instead of the blunting diameter [2]. We will assume that M_∞ of the unperturbed flow is equal to infinity. Then, at fixed $\varkappa$ the dimensionless variables will depend on parameter K_1 alone.

Figure 2 (at $\varkappa = 1.4$) shows the dependences of the value of k (the ratio of the long semiaxis of the ellipse to the small one) and $(X - X_0/X_0$ (the ratio of the resistance of the body without consideration of blunting resistance to the blunting resistance) in function $K_1 = (\pi/2c_x S)^{1/2} L \,\mathrm{tg}^2 \alpha$, in the form of curves, where α is

the half-aperture angle of the round cone. The form of the shock wave was determined from the solution of the problem of flow about a thin bluff cone according to [2]. It is logical to apply the area rule at $X/X_0 \geqslant 1.1$, which, as Fig. 2 indicates, corresponds to $K_1 \geqslant 0.1$. At small values of K_1, the resistance of the body is essentially determined by the value of blunting resistance. At large values of K_1, the area rule loses its validity as k approaches one (more precisely [1], at $k-1 \sim (\varkappa-1)/(\varkappa+1)$), which occurs approximately at $K_1 = 1.2$.

Thus, the range of application of the area rule lies within $0.1 \leqslant K_1 \leqslant 1.2$. Here the ellipse in the cross section of the body can be rather elongated, differing from a circle ($13 \geqslant k \geqslant 1.3$). This result can be of practical interest. However, whereas results [1] were obtained with approximate assumptions about the concentration of the entire mass of gas in the infinitely thin layer beyond the shock wave, it is necessary to make the area rule more precise.

2. Refining the Area Rule. As before, we will assume that $M_\infty \gg 1$, but, unlike in [1], we will not impose the necessary condition $M_\infty \tau \gg 1$. We will introduce cylindrical coordinate system xL, yL, θ (the x-axis passes through the front point of the body and is directed along the flow). We will use uU_∞, vU_∞, wU_∞ to designate

the velocity components in the axial, radial, and circumferential directions, respectively; $p\rho_\infty U_\infty^2$ — is pressure; $\rho\rho_\infty$ — is density; U_∞ is the velocity of the unperturbed flow directed along the x-axis; and ρ_∞ is its density. We will characterize the blunting value by the dimensionless blunting diameter dL, where d is a small value. We will write the equation for the surface of the body in the form $y = r(f)(x, \theta)$.

We will isolate the entropy layer, i.e., the region occupied by the current lines passing through that section of the surface of the retreating shock wave where the angles of inclination formed by the surface of the shock wave and the direction of the unperturbed flow are not small (Fig. 3). Suppose that the equation of the arbitrarily introduced boundary of the entropy layer is $y = \delta\phi(x, \theta)$, where δ is small. Beginning with a certain $x = x_0 \sim d$, the angles of inclination of the boundary of the entropy layer to the x-axis will be on the order of δ.

Below we will give estimates of the parameters of flow in the entropy layer similar to those made in [3]. We will also point out that the effect of the entropy layer on the pressure distribution on a thin bluff cone was considered in [4].

We will assume that the relationship $\tilde{p} \sim \bar{d}^{a}$, holds on the

surface of the entropy layer, where α is the positive number to be determined.

As follows subsequently, the order of pressure is maintained across the entropy layer; therefore, with the condition of adiabaticity, we can write $\rho \sim d^{\frac{\alpha}{\varkappa}}$. for density. Now we will write the continuity equation of the entropy layer. Equating the flow in the entropy layer to the flow in the jet current of unperturbed flow through an area equal to that of the maximum midsectional blunting area, we will have

$$d^2 \sim \rho u \sigma, \quad (1)$$

where σ is the area occupied by the entropy layer in cross section x = const (the shaded part in Fig. 3). As follows from the Bernoulli equation, we will have $\sigma \sim d^{2-\frac{\alpha}{\varkappa}}$. since u ~ 1 in the entropy layer. It is obvious that estimate

$$\delta^2 = S + \sigma, \quad (2)$$

is obtained for δ in the entropy layer boundary equation, where S is the cross-sectional area of the body. We will stipulate that the order of magnitude of the area of the body does not exceed the area of the entropy layer $S \lesssim \sigma$. Then, obviously,

$$\delta^2 \sim d^{2-\frac{\alpha}{\varkappa}}. \quad (3)$$

Since the usual estimate of hypersonic flow $p \sim \delta^2$ is valid for the pressure on the external boundary of the entropy layer, we will find the equation for determining α from it:

$$d^{\alpha} \sim d^{2-\frac{\alpha}{\varkappa}}, \quad \alpha = \frac{2\varkappa}{\varkappa+1} \tag{4}$$

Finally, we will have the following flow parameters in the entropy layer:

$$p \sim d^{\frac{2\varkappa}{\varkappa+1}}, \quad \rho \sim d^{\frac{2}{\varkappa+1}}, \quad \delta \sim d^{\frac{\varkappa}{\varkappa+1}}, \quad \sigma \sim d^{\frac{2\varkappa}{\varkappa+1}} \tag{5}$$

with the condition that the orders of magnitude of τ and d are related by the relationship obtained with the condition S ~ σ:

$$\tau \sim d^{\frac{\varkappa}{\varkappa+1}} \tag{6}$$

This relationship essentially conforms to the condition $\tau \sim \sqrt{d}$, which says that the order of magnitude of the resistance of the body is comparable to the blunting resistance [2].

We will estimate the pressure gradient in the entropy layer.

From the equations of motion we have

$$\frac{\partial p}{\partial y} = -\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{w}{y}\frac{\partial v}{\partial \theta}\right), \tag{7}$$
$$\frac{1}{y}\frac{\partial p}{\partial \theta} = -\rho\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + \frac{w}{y}\frac{\partial w}{\partial \theta}\right).$$

Therefore, $\frac{\partial p}{\partial y} \sim \frac{1}{y}\frac{\partial p}{\partial \theta}$, since all the terms in the right sides of (7) are of the same order of smallness. Whence, with consideration of (5) and (6), the estimate

$$\Delta p \sim d^2 \sim \tau^{\frac{2(\varkappa+1)}{\varkappa}}.$$

is valid for the pressure gradient in both the radial and circumferential directions. Thus, the pressure in the entropy layer can be considered to be constant with the relative error

$$\frac{\Delta p}{p} \sim d^{\frac{2}{\varkappa+1}} \sim \tau^{\frac{2}{\varkappa}} \quad (8)$$

which is somewhat greater than the relative error in the theory of small perturbations of the hypersonic flow (equal, as we know, to τ^2).

We will state the flow problem. Suppose that at $x < x_0$ the axisymmetrical nose section of a body is given with its axis of

symmetry directed along the x-axis, the flow about which has been completely calculated. As follows from the above discussion, since the entropy layer cannot maintain the pressure gradient in the circumferential direction, the pressure on cross section x = const should be constant in region $x > x_0$ on the outer edge of the layer. It suffices for the surface which bounds the entropy layer to be axially symmetrical in order to satisfy this condition. (Here its equation can be written as $y = \delta Y(x)$.)

Then the flow outside the entropy layer, which is axisymmetrical according to condition at $x < x_0$, also retains its axial symmetry at $x > x$. Therefore, the condition of the constancy of the pressure in the circumferential direction on the external boundary of the entropy layer will be satisfied.

We will single out the equation which relates S and σ at $x > x_0$, for which we will use the continuity equation, as in [3]. We will use the subscript 0 to designate the values in plane x_0. Isolating the elementary current jet in the entropy layer, we will write its flow equation:

$$\rho_0 u_0 y_0 d\theta_0 dy_0 = \rho u y d\theta dy. \qquad (9)$$

We will have the following for ρ and u from the adiabatic and Bernoulli equations, in which the terms on the order of τ^2 have been

eliminated:

$$\rho=\rho_0\left(\frac{p}{p_0}\right)^{\frac{1}{\varkappa}};\quad \frac{u^2}{2}+\frac{\varkappa}{\varkappa-1}\cdot\frac{p_0}{\rho_0}\left(\frac{p}{p_0}\right)^{\frac{\varkappa-1}{\varkappa}}=\frac{1}{2}+\frac{1}{(\varkappa-1)M_\infty^2}. \tag{10}$$

We will divide equation (9) by ρu and integrate over the entire entropy layer in cross section $x = x_0$ (we will use the same condition: that the coordinates of the selected current line y and θ satisfy relationships $y = y(y_0, \theta_0)$, $\theta = \theta(y_0, \theta_0)$). Obviously, on the right side of the equation we will find area σ occupied by the entropy layer in cross section x. With consideration of the axial symmetry of the boundary of the entropy layer, we will have $\sigma = \pi\delta^2Y^2(x) - S$. Finally, the unknown relationship is written as:

$$\pi\delta^2Y^2(x)-F(p)=S(x),$$

$$F(p)=\iint\limits_{\sigma_0}\left(\frac{p_0}{p}\right)^{\frac{1}{\varkappa}}\sqrt{\frac{1+\frac{2}{\varkappa-1}\frac{1}{M_\infty^2}-\frac{2\varkappa}{\varkappa-1}\frac{p_0}{\rho_0}}{1+\frac{2}{\varkappa-1}\frac{1}{M_\infty^2}-\frac{2\varkappa}{\varkappa-1}\frac{p_0}{\rho_0}\left(\frac{p}{p_0}\right)^{\frac{\varkappa-1}{\varkappa}}}}\,y_0d\theta_0dy_0. \tag{11}$$

Axisymmetrical flow outside the entropy layer at $x > x_0$ can be calculated by one of the known point methods, e.g., the characteristics method. Here the boundary of the entropy layer will be determined from equation (11), which plays a role in the boundary condition which replaces the nonflow condition, during the solution. Whence it follows that the flow is completely determined by the assignment of the law of the change in the cross-sectional area of the body S(X) at $x > x_0$. Here one more obvious limitation must be

imposed on the shape of the body: the body must not go beyond the limits of the "entropy circle" (see Fig. 3), which can be written symbolically as:

$$S \subset \pi\delta^2 Y^2(x). \tag{12}$$

Now we can state the exact hypersonic area rule as follows. During flow about thin bluff bodies with axisymmetrical nose sections which coincide at a certain distance from the front point of the body and identical laws of the change in the cross section of the remaining parts:

a) the flows outside the entropy layers are axisymmetrical; the flow parameters at the corresponding points, the surfaces of the shock waves, and the arbitrarily introduced boundaries of the entropy layers coincide;

b) the pressure in the entropy layers only depends on x, and the law of the change in pressure is the same for the bodies in question; as a result, the drags acting on the bodies are equal, since drag X is expressed as

$$X = X_0 + L^2\rho_\infty U_\infty^2 \int_{x_0}^{1} S'(x)\, p(x)\, dx, \tag{13}$$

where X_0 is the drag of the nose section of the body. Here it is

assumed that conditions (6) and (12) are satisfied.

The results obtained can easily be generalized to the case of flow with dissociation. The consideration of these phenomena only changes the appearance of function F(p) in equation (11).

3. Comparison of Results. We will compare the result obtained with the correct area proven in [1]. The requirement of the correspondence of the laws of the change in the cross-sectional area in the direction of the x-axis, as well as the condition which states that the drag of the body must not exceed the order of value of the blunting resistance, are common to both theorems. The difference in the formulation of the theorem proven in §2 is as follows:

a) the necessary condition $M_\infty \tau \gg 1$; is not imposed, as was done in [1];

b) instead of requiring the equivalence of the values of the blunting resistance [1], a greater limitation is imposed: the nose sections of the equivalent bodies, being axisymmetrical, must have the same shape;

c) instead of condition (3) in [1], which states that the body

cannot go beyond the limits of the space bounded by the surface of the shock wave, a greater limitation (12) is imposed: the body must not go beyond the upper boundary of the entropy layer.

For this reason, we can expect the values of k found in §1, characterizing the difference in the cross section of the body from that of the equivalent solid of revolution, to be somewhat higher.

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Fig. 1. 1 - Shock wave; 2 - cross section of round body; 3 - cross section of equivalent body.

Fig. 2.

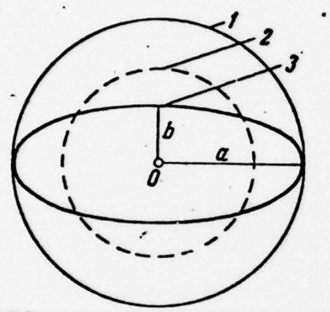


Fig. 1

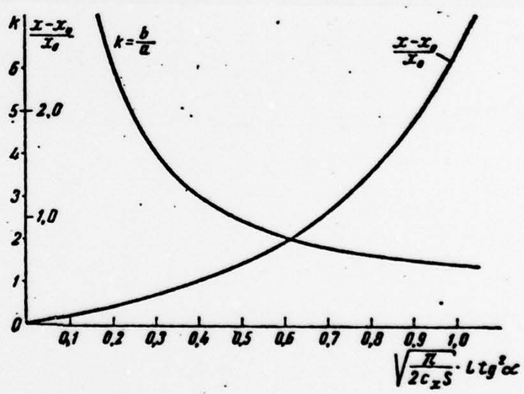
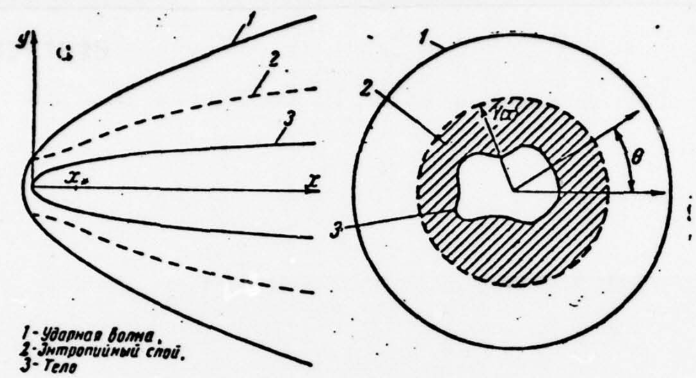


Fig. 2

Fig. 3. 1 - Shock wave; 2 - entropy layer; 3 - body.



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